OCR
Oxford Cambridge and RSA

Monday 26 June 2017 – Afternoon

A2 GCE MATHEMATICS

4726/01 Further Pure Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4726/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **20** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

1 Find the x-coordinate of the stationary point on the curve $y = 3\cosh x - 2\sinh x$, giving your answer exactly in logarithmic form. **[3]**

2 By first completing the square in the denominator, find the exact value of

$$\int_3^4 \frac{1}{x^2 - 6x + 10}\,\mathrm{d}x. \quad \textbf{[4]}$$

3 It is given that $y = \tan^{-1}\left(\frac{x}{x+1}\right)$.

(i) Show that, when $x = 0$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -2$. **[4]**

(ii) Find the Maclaurin's series for $\tan^{-1}\left(\frac{x}{x+1}\right)$ up to and including the term in x^2. **[2]**

4 The equation of a curve is $y = \frac{x-1}{x^2 - x - 2}$.

(i) Write down the equations of the asymptotes. **[3]**

(ii) State the coordinates of the point where the curve cuts one of its asymptotes. **[1]**

(iii) Show that there are no turning points on the curve. **[4]**

(iv) Sketch the curve. **[2]**

5 The polar equations of two curves are $r = \sin\theta$ and $r = \cos 2\theta$.

(i) On the same diagram sketch the parts of both curves that are in the positive quadrant. **[4]**

The curves meet at the origin and also at a point A.

(ii) Determine the polar coordinates of A. **[2]**

(iii) Find the exact area between the two curves in the positive quadrant. **[8]**

6 **(i)** Using the definition of $\sinh x$ in terms of e^x and e^{-x}, show that

$$\sinh 3x = 4\sinh^3 x + 3\sinh x.$$ **[3]**

(ii) Use the substitution $w = \sinh x$ to find the real root of the equation

$$4w^3 + 3w - 3 = 0.$$

Give your answer in the form $a\ln\left(b + \sqrt{c}\right)$ where a, b and c are real numbers to be determined. **[4]**

7 It is given that $I_n = \int_0^1 x^n \sqrt{1-x}\,\mathrm{d}x$ for $n \geqslant 0$.

(i) Show that $I_n = \frac{2n}{2n+3} I_{n-1}$. **[5]**

(ii) Deduce that $I_n < I_{n-1}$. **[2]**

(iii) Show that $I_4 = \frac{256}{3465}$. **[3]**

8 It is required to solve the equation $\mathrm{f}(x) = \ln(4x-1) - x = 0$.

(i) Show that the equation has two roots, α and β, such that $0.5 < \alpha < 1$ and $1 < \beta < 2$. **[1]**

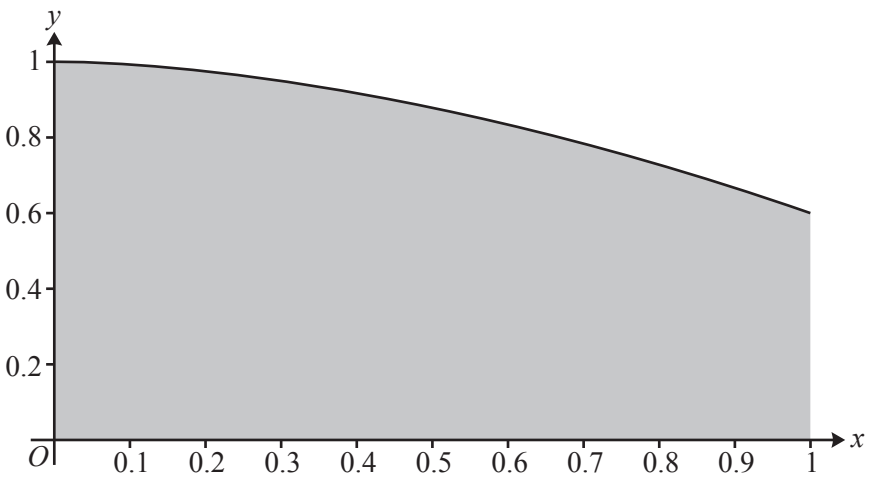
(ii) Use the iterative formula $x_{r+1} = \ln(4x_r - 1)$ with $x_0 = 1.8$ to find x_1, x_2 and x_3, correct to 5 decimal places. Write down the value of β to as many decimal places as these values justify . **[3]**

(iii) Derive the iterative formula $x_{r+1} = \frac{\left(e^{x_r} + 1\right)}{4}$ and use it to find α correct to 4 decimal places. **[4]**

(iv) Show that the iterative formula in part **(ii)** will not find the value of α. Determine whether the iterative formula in part **(iii)** will find the value of β. **[3]**

Question 9 begins on page 4.

9 The diagram shows the curve $y = \mathrm{e}^{-x^2}$ for $0 \leqslant x \leqslant 1$. The region between the curve and the x-axis for $0 < x < 1$ is shaded.



(i) By considering n rectangles of equal width, show that an upper bound, U, for the area of the shaded region is $U = \frac{1}{n}\sum_{r=0}^{n-1} \mathrm{e}^{-\left(\frac{r}{n}\right)^2}$. **[3]**

(ii) By considering another set of n rectangles of equal width, find a similar expression for a lower bound, L, for the area of the shaded region. **[1]**

(iii) Determine the least value of n such that $U - L < 10^{-4}$. **[3]**

END OF QUESTION PAPER