

OCR

Oxford Cambridge and RSA

Monday 26 June 2017 – Afternoon

A2 GCE MATHEMATICS

4726/01 Further Pure Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4726/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **20** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Answer **all** the questions.

- 1 Find the x -coordinate of the stationary point on the curve $y = 3 \cosh x - 2 \sinh x$, giving your answer exactly in logarithmic form. [3]

- 2 By first completing the square in the denominator, find the exact value of

$$\int_3^4 \frac{1}{x^2 - 6x + 10} dx. \quad [4]$$

- 3 It is given that $y = \tan^{-1}\left(\frac{x}{x+1}\right)$.

(i) Show that, when $x = 0$, $\frac{d^2y}{dx^2} = -2$. [4]

(ii) Find the Maclaurin's series for $\tan^{-1}\left(\frac{x}{x+1}\right)$ up to and including the term in x^2 . [2]

- 4 The equation of a curve is $y = \frac{x-1}{x^2-x-2}$.

(i) Write down the equations of the asymptotes. [3]

(ii) State the coordinates of the point where the curve cuts one of its asymptotes. [1]

(iii) Show that there are no turning points on the curve. [4]

(iv) Sketch the curve. [2]

- 5 The polar equations of two curves are $r = \sin \theta$ and $r = \cos 2\theta$.

(i) On the same diagram sketch the parts of both curves that are in the positive quadrant. [4]

The curves meet at the origin and also at a point A .

(ii) Determine the polar coordinates of A . [2]

(iii) Find the exact area between the two curves in the positive quadrant. [8]

- 6 (i) Using the definition of $\sinh x$ in terms of e^x and e^{-x} , show that

$$\sinh 3x = 4 \sinh^3 x + 3 \sinh x. \quad [3]$$

- (ii) Use the substitution $w = \sinh x$ to find the real root of the equation

$$4w^3 + 3w - 3 = 0.$$

Give your answer in the form $a \ln(b + \sqrt{c})$ where a , b and c are real numbers to be determined. [4]

- 7 It is given that $I_n = \int_0^1 x^n \sqrt{1-x} \, dx$ for $n \geq 0$.

(i) Show that $I_n = \frac{2n}{2n+3} I_{n-1}$. [5]

(ii) Deduce that $I_n < I_{n-1}$. [2]

(iii) Show that $I_4 = \frac{256}{3465}$. [3]

- 8 It is required to solve the equation $f(x) = \ln(4x-1) - x = 0$.

(i) Show that the equation has two roots, α and β , such that $0.5 < \alpha < 1$ and $1 < \beta < 2$. [1]

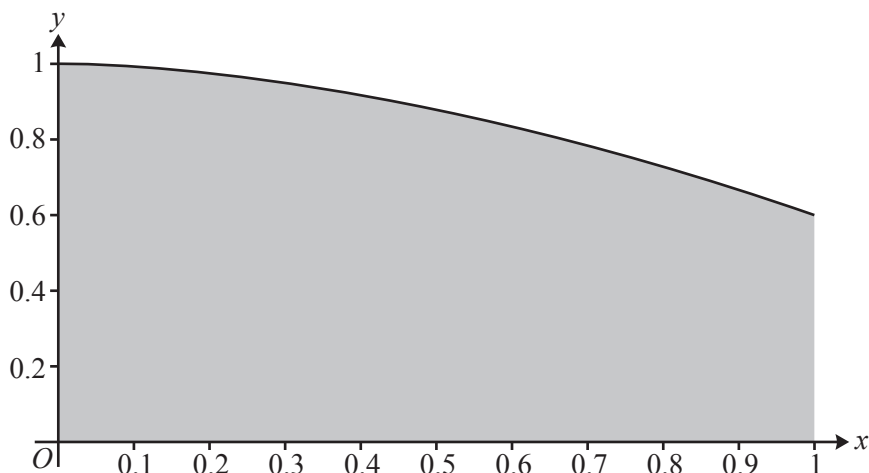
(ii) Use the iterative formula $x_{r+1} = \ln(4x_r - 1)$ with $x_0 = 1.8$ to find x_1 , x_2 and x_3 , correct to 5 decimal places. Write down the value of β to as many decimal places as these values justify. [3]

(iii) Derive the iterative formula $x_{r+1} = \frac{(e^{x_r} + 1)}{4}$ and use it to find α correct to 4 decimal places. [4]

(iv) Show that the iterative formula in part (ii) will not find the value of α . Determine whether the iterative formula in part (iii) will find the value of β . [3]

Question 9 begins on page 4.

- 9 The diagram shows the curve $y = e^{-x^2}$ for $0 \leq x \leq 1$. The region between the curve and the x -axis for $0 < x < 1$ is shaded.



- (i) By considering n rectangles of equal width, show that an upper bound, U , for the area of the shaded region is $U = \frac{1}{n} \sum_{r=0}^{n-1} e^{-\left(\frac{r}{n}\right)^2}$. [3]
- (ii) By considering another set of n rectangles of equal width, find a similar expression for a lower bound, L , for the area of the shaded region. [1]
- (iii) Determine the least value of n such that $U - L < 10^{-4}$. [3]

END OF QUESTION PAPER

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